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HYDRODYNAMICS OF CLOUD SYSTEMS

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SUMMARY

This paper describes a particular case of problems of hydrodynamic theory of cloud systems applying it to two cloud models, and deriving the respective formulas.

* * *

One of the current problems of dynamic meteorology is the creation of a detailed hydrodynamic theory of separate clouds and cloud systems. When creating such a theory, one must take into account that independently from, whether or not the cloud shifts, there takes place inside it an intensive air motion. This motion may be traced as pseudoadiabatic, while the motion outside the cloud may be considered in our problem as adiabatic. The boundary of the cloud is not known in advance and it must be determined alongside with the solution of the problem itself.

Let us limit ourselves at the moment to the case of a "fixed" cloud, and let us consider the stage of development of the cloud, when the process may be considered as stationary, that is the stage of developed cloud. The motion, concomitant to the cloud, will be considered as axisymmetrical. The equations of motion will be taken in the form (the system of coordinates being cylindrical) :

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* K GIDRODINAMIKE OBLACHNYKH SISTEM.

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial\Phi}{\partial r}, \qquad (1)$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=0; \qquad (2)$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial\Phi}{\partial z}+\frac{g}{T_m}T', \qquad (3)$$

$$\frac{\partial r\rho_\infty u}{\partial r}+\frac{\partial r\rho_\infty w}{\partial z}=0; \qquad (4)$$

$$u\frac{\partial T'}{\partial r}+w\frac{\partial T'}{\partial z}+(\varepsilon\gamma_a-\gamma)w=0. \qquad (5)$$

Here z is the altitude above the Earth's surface; r is the distance from the axis of symmetry; u, v, w are respectively the velocity components along the axis r, the circle and the vertical; T' is the temperature T deflection from its value $T_\infty(z)$ at great distance from the cloud; g is the gravitation acceleration ; T_m is the mean temperature of the air column; Φ is the geopotential's deflection from its standard value; γ_a is the adiabatic gradient ; $\gamma=-dT_\infty/dz=\text{const}$; $\varepsilon=1$ beyond the cloud;

$$\varepsilon=\varepsilon_\infty=\left[1-\frac{0{,}623}{c_p}L\frac{k}{k-1}\frac{l_{max}(T_\infty)}{p_\infty T_\infty}\right]\left[1+\frac{0{,}623}{c_p}\frac{L}{p_\infty}\frac{dl_{max}}{dT_\infty}\right]^{-1},$$

with, at the same time,

$$l_{max}(T_\infty)=6{,}1\cdot10^{-3}P\exp\left(17{,}13\,\frac{T_\infty-273}{T_\infty-38}\right)$$

[see (1)], where c_p is the heat capacity of the air at constant pressure; k - the heat capacities' ratio; L is the latent condensation heat *; $p_\infty(z)$ is the standard pressure; P is the pressure at sea level; $\rho_\infty(z)$ is the standard density.

The system (1)- (5) has four integrals which may be constructed after the current function ψ has been introduced by way of (4) from the equalities

$$r\frac{\rho_\infty(z)}{\rho_\infty(0)}u=-\frac{\partial\psi}{\partial z}; \quad r\frac{\rho_\infty(z)}{\rho_\infty(0)}w=\frac{\partial\psi}{\partial r}. \qquad (6)$$

* Only the presence of the liquid phase is assumed. The generalization to the case when the solid phase is also present may be made easily.

Then, according to (5), we may write

$$T' = -(\gamma_a - \gamma)[z - f_1(\psi)] \qquad \text{outside the cloud} \quad (7)$$

$$T' = -(\gamma_a - \gamma)\left[\int_0^z \frac{\varepsilon_\infty \gamma_a - \gamma}{\gamma_a - \gamma} dz - \tilde{f}_1(\psi)\right] \qquad \text{inside the cloud} \quad (8)$$

where f_1 and $\bar{f}_1$ are arbitrary functions of ψ; finally, by way of exclusion of Φ, we shall obtain outside the cloud

$$\frac{1}{\bar{\rho}} \frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2 \bar{\rho}} \frac{\partial}{\partial z}\left(\frac{1}{\bar{\rho}} \frac{\partial \psi}{\partial z}\right) = -\frac{f_2}{r^2} \frac{df_2}{d\psi} + \frac{g(\gamma_a - \gamma)}{T_m}\left[f_3(\psi) + z \frac{df_1}{d\psi}\right] \quad (9)$$

($\bar{\rho} = \rho_\infty(z) : \rho_\infty(0)$), where f_3 is a new arbitrary function of ψ, and an analogous equation for the cloud with substitution of f_3 by $\bar{f}_3$ (new function). The fourth integral, that is the Bernoulli law, is not written out.

In the following we shall assume that the boundary of the cloud will be determined by one of two conditions: either this is the surface, intersected by the current lines and which separates the region of saturation from that, where vapors do not saturate the space ("lower" boundary), or it is the surface of the current, on which ψ = const. ("upper" boundary). On the lower boundary, we shall write the saturation conditions, which will be represented in the form [see (1)]:

$$z_{\text{кр}} = m f_1(\psi_{\text{кр}}) + s, \quad (10)$$

where

$$m = [\Gamma + (\gamma_a - \gamma) b](b\gamma_a - g / RT_m)^{-1}; \quad (11)$$

$$s = (bt_0 + \ln(0{,}38P / q_0 p_0))(b\gamma_a - g / RT_m)^{-1}, \quad (12)$$

at the same time $b = 20{,}2 : T_m; t_0 = T_0 - 273$; T_0 is the temperature at $z = 0$ far off the cloud; we then estimate that the specific moisture q_∞ far off the cloud is given by the function $q_\infty = q_0 \exp(-\Gamma z)$, where q_0 and Γ are constants.

Aside from (10), we shall require at the lower boundary: a) the continuity of temperature transition; b) the continuity of vortex transition $\partial w / \partial r - \partial u / \partial z$; c) the continuity of ψ transition; d) the continuity of $\partial \psi / \partial z$ transition.

The condition a) allows, by (7), (8) and (11), to link $\tilde{f}_1$ and f_1:

$$\tilde{f}_1(\psi) = (1-m) f_1 + \int_0^{mf_1+s} \frac{e_\infty \gamma_a - \gamma}{\gamma_a - \gamma} dz - s. \qquad (13)$$

The condition b) will interrelate f_3 and f_3 by (9) and (13):

$$\tilde{f}_3(\psi) = f_3(\psi) + (mf_1 + s) mn\, df_1 / d\psi, \qquad (14)$$

where

$$n = 1 - (e_\infty \gamma_a - \gamma) / (\gamma_a - \gamma) = (\gamma_a - e_\infty \gamma_a) / (\gamma_a - \gamma) \qquad (15)$$

(<u>n</u> may be practically considered as constant).

Let us now rewrite (9) in the final form, introducing the dimensionless quantities $\psi, \bar{z}, \bar{r}, \bar{u}, \bar{v}, \bar{w}, f, S, F, \Phi$ from the equalities

$$\psi_0 \bar{\psi} = \psi, \quad H\bar{z} = z, \quad H\bar{r} = r, \quad fH = f_1, \quad UHF = f_2,$$
$$H^2 \Phi = f_3 \psi_0, \quad HS = s, \quad U\bar{u} = u, \quad U\bar{v} = v, \quad U\bar{w} = w,$$
$$\psi_0 = H^3 \sqrt{g(\gamma_a - \gamma) / T_\infty}, \quad U = \psi_0 / H^2 \,*; \quad H = RT_m / g.$$

At the same time, we estimate that $\bar{\rho} = \exp(-\bar{z})$, and we shall introduce moreover $\zeta = 1 - e^{-\bar{z}}$. We shall obtain, dropping the strokes above the letters,

$$\frac{1}{\bar{\rho}} \frac{1}{r} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \zeta^2} = \Phi + z \frac{df}{d\psi} - \frac{1}{r^2} F \frac{dF}{d\psi}; \qquad (16)$$

$$\frac{1}{\bar{\rho}} \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \zeta^2}$$
$$= \Phi + \frac{df}{d\psi} [m^2 nf + mnS + (1 - mn) z] - \frac{1}{r^2} F \frac{dF}{d\psi}. \qquad (17)$$

It remains to select f, F and Φ.

Let us limit ourselves to the examination of two models: a cumulus cloud model, and a model of the central part of a typhoon.

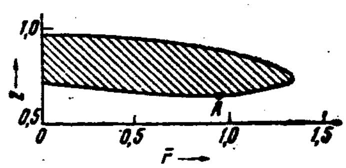

Fig. 1

For a cumulus cloud F = 0. We choose

$$\Phi = -f\, df / d\psi, \quad f = 1/2 - 1/2 \sqrt{1 + 4\psi}$$

under the cloud, and

$$f = -1/2 + 1/2 \sqrt{1 + 4\psi}$$

above the cloud.

The boundary of a stationary cloud, responding to the solution of (16) and (17) for m = 4, n = 2.25, S^2 = 0.5, is schematically shown in Fig.1.

At computations we assume $\rho = 1$, and at the same time we bound the atmosphere from above by a wall of height h (" inversion"); h replaces the quantity RT_m / g. To the right of the point A we have descending currents, to the left — the ascending ones. The shape of the clouds reminds us of the pattern of elongated clouds of good weather.

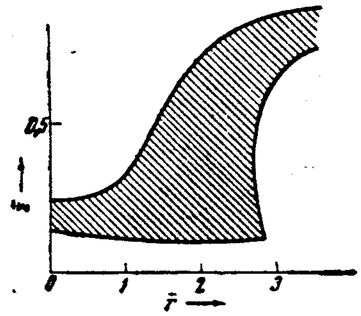

Fig 2

For a typhoon (Fig. 2) we assumed, under the cloud and above the cloud, respectively:

$F = \omega\psi / (\alpha^2 + \psi^2)$, $\quad \Phi = -\int df / d\psi;\ f = \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\psi}$

and

$$f = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\psi}$$

In Fig. 2 we represented schematically the boundary of the cloudiness (approximate solution of (16) and (17) at $m = 0.8$, $n = 1.25$, $S = 0.15$, $\alpha = 1/16$ and $\omega = 3.94$. We again assumed $\rho = 1$, and instead of H we took h (" tropopause"). The shape of the "eye of the storm", of the "anvil" and of certain other details generally agrees well with the observations.

***** THE END *****

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